BACKPAPER EXAMINATION

COMPLEX ANALYSIS
B. MATH III YEAR
I SEMESTER, 2009-2010

Max. Marks: 100 Time Limit: 3 hrs

Notation: U stands for the open unit disk.

1. Find all entire functions f such that $f(z)^{10} = z$ for all z. [10]

2. Let p be a polynomial, Ω a bounded region and suppose the maximum of $|p|$ is attained at an interior point a of Ω. Show without the use of Maximum Modulus principle that p is a constant. Hint: integrate $\frac{p(z)}{z-a}$ over a small circle around a. [15]

3. If $f : \{z : |z| < 2\} \to U$ is holomorphic prove that f has atmost one fixed point. [15]

4. Find the largest open set Ω in $\mathbb{C}$ such that $\prod \frac{z-1/n-1/n^2}{z-1/n}$ is holomorphic in Ω. [10]

5. Let f be the principal branch of the logarithm on $\mathbb{C}\backslash(-\infty, 0]$. Prove that $f(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(z-1)^n$ if $z \in B(1,1)$. [15]

6. Find a holomorphic bijection f from U onto itself such that $f(0) = 1/2$ and $f'(0) = i$. How many such functions are there? [15]

7. Evaluate $\int_{-\pi}^{\pi} \frac{2\sin t}{3+\cos t} dt$ by the method of residues. [20]